## Third Semester B.E. Degree Examination, December 2011 Engineering Mathematics - III

Time: 3 hrs.

Max. Marks:100

Note: 1. Answer any FIVE full questions, selecting at least TWO questions from each part. 2. Missing data will be suitably assumed.

a. Obtain the Fourier series for the function  $f(x) = \begin{cases} \pi x &: 0 \le x \le 1 \\ \pi(2-x) &: 1 \le x \le 2 \end{cases}$  and deduce that

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}.$$
 (07 Marks)

Obtain the half range Fourier sine series for the function.

(07 Marks)

$$f(x) = \begin{cases} 1/4 - x ; & 0 < x < 1/2 \\ x - 3/4 ; & 1/2 < x < 1 \end{cases}$$

Compute the constant term and the first two harmonics in the Fourier series of f(x) given by the following table. (06 Marks)

 $f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \le 1 \\ 0, & \text{for } |x| > 1 \end{cases}$ Find the Fourier transform of and hence evaluate

$$\int_{0}^{\infty} \left( \frac{x \cos x - \sin x}{x^{3}} \right) \cos \frac{x}{2} dx . \tag{07 Marks}$$

- Find the Fourier cosine transform of  $f(x) = \frac{1}{1 + x^2}$ . (07 Marks)
- c. Solve the integral equation  $\int_{0}^{\infty} f(\theta) \cos \alpha \theta d\theta = \begin{cases} 1 \alpha & ; & 0 \le \alpha \le 1 \\ 0 & ; & \alpha > 1 \end{cases}$ . Hence evaluate  $\int_{0}^{\infty} \frac{\sin^{2} t}{t^{2}} dt$ . (06 Marks)
- Solve two dimensional Laplace equation  $u_{xx} + u_{yy} = 0$ , by the method of separation of variables. (07 Marks)
  - Solve the one dimensional heat equation  $\frac{\partial u}{\partial t} = \frac{c^2 \partial^2 u}{\partial x^2}$ ,  $0 < x < \pi$  under the conditions:

i)  $u(0,+) = 0, u(\pi, t) = 0$ ii)  $u(x, 0) = u_0 \sin x$  where  $u_0 = \text{constant} \neq 0$ . (07 Marks)

Obtain the D' Alembert's solution of one dimensional wave equation. (06 Marks)

a. Fit a curve of the form  $y = ae^{bx}$  to the following data: (07 Marks) x: 77 100 185 239

y : 2.4 3.4 7.0 11.1 19.6 Using graphical method solve the L.P.P minimize  $z = 20x_1+10x_2$  subject to the constraints

20

 $x_1+2x_2 \le 40$ ;  $3x_1+x_2 \ge 0$ ;  $4x_1+3x_2 \ge 60$ ;  $x_1 \ge 0$ ;  $x_2 \ge 0$ . c. Solve the following L.P.P maximize  $z = 2x_1 + 3x_2 + x_3$ , subject to the constraints

 $x_1 + 2x_2 + 5x_3 \le 19$ ,  $3x_1 + x_2 + 4x_3 \le 25$ ,  $x_1 \ge 0$ ,  $x_2 \ge 0$ ,  $x_3 \ge 0$  using simplex method. (07 Marks)

### PART - B

- 5 a. Using the Regula falsi method, find the root of the equation  $xe^x = \cos x$  that lies between 0.4 and 0.6. Carry out four iterations. (07 Marks)
  - b. Using relaxation method solve the equations:

10x - 2y - 3z = 205; -2x + 10y - 2z = 154; -2x - y + 10z = 120.

c. Using the Rayleigh's power method, find the dominant eigen value and the corresponding

eigen vector of the matrix.  $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$  starting with the initial vector  $\begin{bmatrix} 1,1,1 \end{bmatrix}^T$ .

(06 Marks)

(07 Marks)

6 a. From the following table, estimate the number of students who have obtained the marks between 40 and 45:

(07 Marks)

Marks : 30-40 40-50 50-60 60-70 70-80 Number of students : 31 42 51 35 31

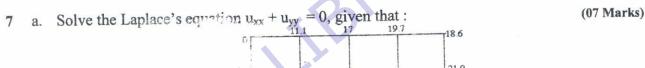
b. Using Lagrange's formula, find the interpolating polynomial that approximate the function described by the following table:

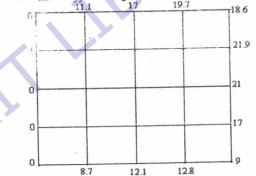
(07 Marks)

c. A curve is drawn to pass through the points given by the following table:

x: 1 1.5 2 2.5 3 3.5 4 y: 2 2.4 2.7 2.8 3 2.6 2.1

Using Weddle's rule, estimate the area bounded by the curve, the x - axis and the lines x = 1, x = 4. (06 Marks)





- b. Solve  $\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$  subject to u(0, t) = 0; u(4, t) = 0; u(x, 0) = x (4 x). Take h = 1, k = 0.5.
- c. Solve the equation  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  subject to the conditions  $u(x, 0) = \sin \pi x$ ,  $0 \le x \le 1$ ; u(0, t) = u(1, t) = 0 using Schmidt's method. Carry out computations for two levels, taking h 1/3, k = 1/36.
- 8 a. Find the Z transform of : i)  $(2n-1)^2$  ii)  $\cos\left(\frac{n\pi}{2} + \pi/4\right)$  (07 Marks)

b. Obtain the inverse Z – transform of  $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$ . (07 Marks)

c. Solve the difference equation  $y_{n+2} + 6y_{n+1} + 9y_n = 2n$  with  $y_0 = y_1 = 0$  using Z transforms. (06 Marks)

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## Third Semester B.E. Degree Examination, December 2011

## Analog Electronic Circuits

Time: 3 hrs.

Max. Marks: 100

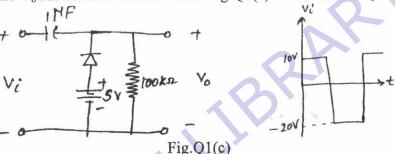
Note: 1. Answer any FIVE full questions, selecting at least TWO questions from each part. 2. Missing data, if any, may be suitably assumed.

#### PART – A

- Explain the following with respect to a semiconductor diode:
  - Diffusion capacitance
  - Transition capacitance and ii)

iii) Reverse recovery time. (06 Marks)

- b. Explain the working of a half wave rectifier. Also determine ripple factor, efficiency and peak inverse voltage. (08 Marks)
- Determine V<sub>0</sub> for the network shown in Fig.Q1(c). Also sketch V<sub>0</sub>.

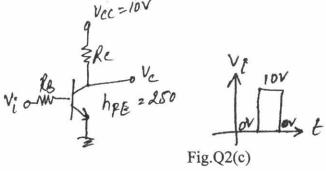


(06 Marks)

- In a voltage divider bias circuit of BJT  $R_C = 4K\Omega$ ,  $R_E = 1.5K\Omega$ ,  $R_1 = 39K\Omega$ ,  $R_2 = 3.9K\Omega$ ,  $V_{CC} = 18V$  and  $\beta = 70$ . Find  $I_{CQ}$  and  $V_{CEQ}$ .
  - In an emitter bias configuration  $I_{CQ} = \frac{1}{2}I_{CSat}$  and  $I_{CSat} = 8mA$ ,  $V_{CC} = 28V$  and  $V_{C} = 18V$ ,

 $\beta = 110$ . Determine  $R_C$ ,  $R_E$ ,  $R_B$  and stability factor  $S(I_{CO})$ .

c. Determine  $R_B$  and  $R_C$  for the transistor inverter of Fig.Q2(c) if  $I_{CSat} = 10$ mA. (06 Marks)



- (06 Marks) Define h-parameters and hence derive h-parameter model of a CE-BJT.
  - Derive expressions for A<sub>i</sub>, A<sub>v</sub>, Z<sub>i</sub> and Z<sub>o</sub> for a voltage divider bias circuit of BJT, using approximate hybrid model of BJT.
  - A voltage source of negligible internal resistance drives a common collector transistor amplifier. The load resistance is 2500  $\Omega$ . The transistor h-parameters are  $h_{ic} = 1000\Omega$ , (08 Marks)  $h_{rc} = 1$ ,  $h_{fc} = -50$  and  $h_{oc} = 25\mu A/v$ . Compute  $A_i$ ,  $A_v$ ,  $Z_i$  and  $Z_o$ .

 Explain the effect of coupling capacitor and bypass capacitor on the low frequency response of a BJT amplifier. (10 Marks)

b. Determine the lower cut off frequency for the voltage divider bias BJT amplifier with  $C_S=10\mu F,~C_C=1\mu F,~C_E=20~\mu F,~R_S=1K\Omega,~R_1=10K\Omega,~R_2=10K\Omega,~R_E=2K\Omega,~R_C=4K\Omega$  and  $R_L=2.2K\Omega,~\beta=100,~r_0=\infty$  and  $V_{CC}=20V.$  (10 Marks)

#### PART - B

5 a. Derive expressions for Z<sub>i</sub> and A<sub>i</sub> for a Darlington emitter follower circuit. (08 Marks)

b. A 2 stage cascaded amplifier system is built with stage voltage gains 25 and 40. Both stages have the same bandwidth of 220 kHz with identical lower cutoff frequency of 500 Hz. Find the overall gain bandwidth product. (06 Marks)

c. Mention the types of feedback connections. For any one type, derive the gain, with feedback and compare it with that without feedback. (06 Marks)

6 a. Explain the operation of a transformer coupled class-A amplifier. (08 Marks)

b. A class-B amplifier using a supply of  $V_{CC} = 30V$  and driving a load of  $16\Omega$ , determine the maximum input power, output power and transistor dissipation. (06 Marks)

c. Explain the causes of distortion in an amplifier. Also define THD. (06 Marks)

7 a. Explain Barkhausen criterion for oscillation. Also give the classification of oscillators.

(06 Marks)

b. Explain the working of Wien bridge oscillator.

(07 Marks)

c. Explain the working of BJT Colpitt's oscillator.

(07 Marks)

8 a. Derive expression for  $V_{GSQ}$ ,  $I_{DQ}$ ,  $V_{DS}$ ,  $V_S$ ,  $V_G$  and  $V_D$  for a self bias FET circuit. (10 Marks)

b. Explain the depletion and enhancement type MOSEFTs, their characteristics and frequency response. (10 Marks)

## Third Semester B.E. Degree Examination, December 2011 Logic Design

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

#### PART - A

- 1 a. Expand  $f_1 = a + bc + a c d$  into minterms and  $f_2 = a (b + c) (a + c + d)$  into maxterms.

  (06 Marks)
  - b. Simplify  $f(a, b, c, d) = \sum m(1, 2, 4, 11, 13, 14, 15) + dc(0, 5, 7, 8, 10)$  using Karnaugh map technique. (05 Marks)
  - c. Obtain a minimal SOP expression for the function  $f(a, b, c, d, e) = \sum m(3, 7, 11, 12, 13, 14, 15, 16, 18) + dc (24, 25, 26, 27, 28, 29, 30, 31) using Karnaugh map method. (05 Marks)$
  - d. Explain canonical form of Boolean equations with an example. (04 Marks)
- 2 a. Minimize  $f(a, b, c, d) = \pi(0, 6, 7, 8, 9, 13) + \pi dc(5, 15)$  using quine Mc cluskey method. (12 Marks)
  - b. Simplify  $f(a, b, c, d) = \sum m(2, 3, 4, 5, 13, 15) + dc(8, 9, 10, 11)$  taking least significant bit as map entered variable. (08 Marks)
- 3 a. Design and implement a 4 bit look ahead carry adder. (14 Marks)
  - b. Implement 16:1 multiplexer using 4:1 multiplexers. (06 Marks)
- 4 a. Design and implement a 2 BIT digital comparator. (09 Marks)
  - b. Implement a full subtractor using 3-8 line decoder with the decoder having high outputs and active low enable thermal. (05 Marks)
  - c. Implement the Boolean function  $f(a, b, c, d) = \sum m(0, 1, 5, 6, 7, 9, 10, 15)$  using multiplexer with a, b connected to select lines  $s_1, s_0$ . (06 Marks)

#### PART - B

- a. Give the NAND NAND implementation of a gated SR latch with preset and clear facilities, such that when preset = 0, the output should be 1 while clear = 0, the output be 0. Give the truth table clearly indicating gate, clear, preset and input signals and the corresponding outputs.
  - b. Explain the working of a pulse triggered JK master slave flip flop with a truth table.

(06 Marks)

c. Explain the functioning of positive edge triggered D - flip flop. (07 Marks)

- 6 a. Explain 4 bit universal shift register using negative edge triggered D flip flops. (08 Marks)
  - b. Give the circuit of a 4 bit JOHNSON counter using negative edge triggered D flip flops. Draw the timing waveforms with respect to clock starting with an initial state of  $Q_3Q_2Q_1Q_0 = 0000$ . What is the modulus of this counter? (08 Marks)
  - c. What is meant by triggering of flip flops? Name the different triggering methods. (04 Marks)
- 7 a. Compare synchronous and ripple counters.

(03 Marks)

- b. Draw the circuit of a 3 BIT, asynchronous, down counter using negative edge triggered JK flip flops and draw the timing waveforms. (05 Marks)
- c. Design and implement a synchronous counter to count the sequence 0 3 2 5 1 0 using negative edge triggered JK flip flops. (12 Marks)
- 8 a. Explain Mealy and Moore machine models.

(06 Marks)

b. Construct the excitation table, transition table, state table and state diagram for the Moore circuit shown in Fig.Q.8(b). (14 Marks)

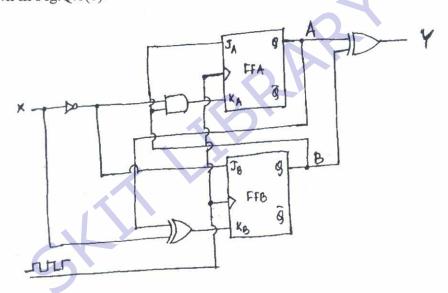


Fig.Q.8(b)

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## Third Semester B.E. Degree Examination, December 2011

## **Network Analysis**

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

#### PART-A

1 a. Using the mesh current method, determine  $V_2$  that results zero current in  $4\Omega$  resistor.

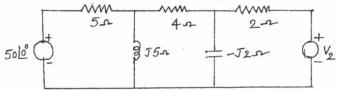
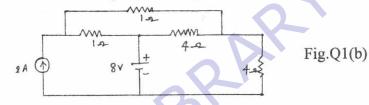


Fig.Q1(a)

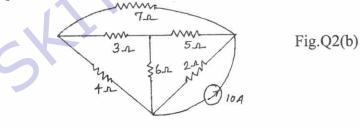
(10 Marks)

b. Find the currents in all resistors by Node voltage method.

(10 Marks)

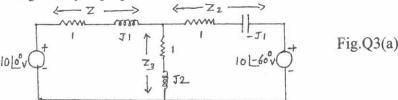


- 2 a. Define the terms: i) Tree, ii) Co-tree, iii) Tie set schedule, iv) Cut set schedule, with respect to a graph of a network. (04 Marks)
  - b. For the network shown, draw the graph, select a tree, write the tie set schedule and obtain the equilibrium equations. Hence currents in various branches. (16 Marks)



3 a. Find the current through  $Z_3$  by superposition theorem.

(10 Marks)

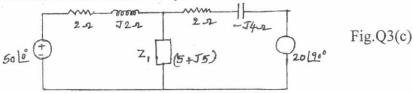


b. State and explain reciprocity theorem.

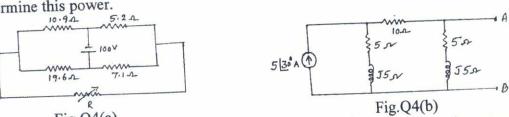
(04 Marks)

c. Use Millman's theorem to find current in Z<sub>1</sub>:

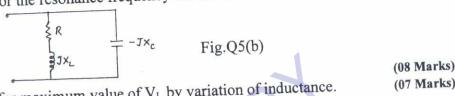
(06 Marks)



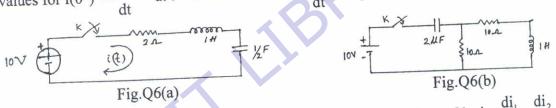
For the circuit shown in Fig.Q4(a), find the value of R that will receive maximum power. Determine this power.



- Obtain the Thevenin and Norton equivalent circuits at terminals AB for the network shown. Hence, find the current through  $10\Omega$  resistor across AB.
  - PART B ii) Q factor, iii) Half power frequency, iv) Band width, Define the terms: i) Resonance, v) Selectivity pertaining to a series RLC circuit.
- Obtain an expression for the resonance frequency for the circuit shown in Fig.Q5(b).



- Obtain the condition for maximum value of V<sub>L</sub> by variation of inductance. C.
- In the network shown, switch 'K' is closed at t = 0 with the capacitor uncharged. Find the 6 values for  $i(0^+)$   $\frac{di(t)0^+}{dt}$  at  $t = 0^+$  and also find  $\frac{d^2i(0^+)}{dt^2}$ (10 Marks)

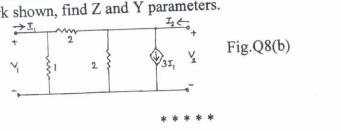


- In the given circuit, switch K is closed at time t = 0. Find the values of  $i_1$ ,  $i_2$ ,  $\frac{di_1}{dt}$ ,  $\frac{di_2}{dt}$ ,  $\frac{d^2i_1}{dt^2}$ , (10 Marks)  $\frac{d^2i_2}{dt^2} \text{ at } t = 0^+.$
- Find the Laplace transform of the following:

5

Find the Laplace transform of the following:  
i) 
$$\sin^2 t$$
 ii)  $\cos^2 t$  iii)  $\sin wt$  iv)  $\int_0^t i(t) dt$  (08 Marks)

- Find the inverse Laplace transform : i)  $\frac{s^2+5}{s(s^2+2s+4)}$ , ii)  $\frac{2s+6}{s^2+6s+25}$ . (08 Marks)
- (04 Marks) State and prove initial value theorem.
- (06 Marks) Express Z parameters in terms of h parameters. (14 Marks) For the network shown, find Z and Y parameters.



(06 Marks)

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# Third Semester B.E. Degree Examination, December 2011 Electronic Instrumentation

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

		PART - A	
1		Explain the following in brief:  i) Gross errors; ii) Relative errors; iii) Accuracy; iv) Resolution.  Explain the working of a true RMS voltmeter with the help of a suitable block diagonal.	(08 Marks) gram. (08 Marks)
	c.	Define sensitivity. Determine the value of the multiplier resistance on the 50V ran voltmeter that uses a 250 μA meter movement with an internal resistance of 100Ω.	ge of a dc
2	a. b.	With block diagram, explain the principle and operation of digital frequency meter	(10 Marks) (10 Marks)
3	a. b.	Explain the CRT features briefly.  Draw the basic block diagram of an oscilloscope. Explain the functions of each block.	(05 Marks) ock. (10 Marks)
	c.	Describe the following modes of operation available in a dual trace oscilloscope: i) ALTERNATE mode; ii) CHOP mode.	(05 Marks)
4	a. b.	Explain why time delay is necessary in oscilloscopes.  Explain the principle and operation of sampling oscilloscope with relevant block	(UO IVIAI NO)
	c.	Explain the operation of digital storage oscilloscope with the help of a block Mention the advantages.	k diagram. (08 Marks)
5	a.	PART – B With block diagram, explain conventional standard signal generator. M	ention the
		applications.  Explain the operation of a function generator with the help of a block diagram.	(10 Marks) (10 Marks)
6	a.	Explain the Wheatstone bridge and derive the balance equation for Wheatstone Mention the limitations.	one bridge. (08 Marks)
	b.	Find the equivalent parallel resistance and capacitance that causes a wein bridge the following component values:	to null with
	c	$R_1 = 3.1 \text{ k} \Omega$ , $C_1 = 5.2 \mu\text{F}$ , $R_2 = 25 \text{ k} \Omega$ , $f = 2.5 \text{ kHz}$ and $R_4 = 100 \text{ k}\Omega$ .	(06 Marks) (06 Marks)
7	a	. What are the factors to be considered for the selection of better transducer? Expla	(no Marks)
	b	Explain the construction, principle and operation of LVDT.	(12 Marks)
8	а	Explain piezo electric transducer, with circuit diagram.	(08 Marks) (06 Marks)

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Compare LED and LCD types of displays.

Write a short note on signal conditioning system.



## Third Semester B.E. Degree Examination, December 2011 Field Theory

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting atleast TWO questions from each part.

#### PART - A

- a. Define electric field intensity due to point charge in a vector form. With usual notations derive expressions for field at a point due to many charges. (06 Marks)
  - b. State and prove Gauss's law.

(06 Marks)

- c. Given  $\vec{D} = 30e^{-r}\hat{a}_r 2z\hat{a}_z$  c/mt<sup>2</sup>. Verify divergence theorem for the volume enclosed by r = 2, z = 5.
- 2 a. Derive an expression for energy and energy density in an electrostatic field. (04 Marks)
  - b. A 15 nc point charge is at the origin in free space. Calculate  $v_1$  if point P is located at P(-2, 3, -1) and : i) V = 0 at (6, 5, 4) ii) V = 0 at infinity. (08 Marks)
  - c. If  $\vec{E} = -8xy\hat{a}_x 4x^2\hat{a}y + \hat{a}_z$  v/m, find the work done in carrying a 6C charge from A(1, 8, 5) to B(2, 18, 6) along the path y = 3x + 2, z = x + 4. (08 Marks)
- 3 a. Starting with point form of Gauss law deduce Poisson's and Laplace's equations. (06 Marks)
  - b. Use Laplace's equation to find the capacitance per unit length of a co-axial cable of inner radius 'a' m and outer radius 'b' m. Assume  $v = v_0$  at r = a and v = 0 at r = b. (08 Marks)
  - c. Determine whether or not the potential equations:

 $V = 2x^2 - 4y^2 + z^2$  ii)  $V = r^2 \cos \phi + \theta$  iii)  $V = r \cos \phi + z$  satisfy the Laplace's equation.

(06 Marks)

- a. Starting from Biot-Savart law, derive an expression for the magnetic field intensity at a point due to finite length of current carrying conductor. (06 Marks)
  - b. Calculate the value of vector current density at P(1.5, 90°, 0.5) if  $\vec{H} = \frac{2}{r}\cos 0.2 \phi \hat{a}_r$ .

(04 Marks)

c. Evaluate both sides of the Stoke's theorem for the field  $\vec{H} = 6xy \, \hat{a}_x - 3y^2 \, \hat{a}_y$  A/m and the rectangular path around the region  $2 \le x \le 5, -1 \le y \le 1, z = 0$ . (10 Marks)

#### PART - B

- 5 a. Obtain boundary conditions at the interface between two magnetic materials. (06 Marks)
  - b. A circular loop of 10 cm radius is located in xy plane with magnetic field  $\vec{B} = 0.5\cos(377t)[3\hat{a}_y + 4\hat{a}_z]T$ . Calculate the voltage induced by the loop. (06 Marks)
    - c. A single trun circular coil 5 cm diameter carries a current of 2.8 A. Determine the magnetic flux density  $\vec{B}$  at a point on the axis 10 cm from the center. Derive the formula used.

      (08 Marks)

(06 Marks)

- 6 a. What is displacement current and equation of continuity? Derive Maxwell's equation for (06 Marks)
  - b. Determine whether or not the following pairs of fields satisfy Maxwell's equation.

$$\vec{E} = E_m \sin x \sin t \hat{a}_y \quad v/m$$

 $\vec{H} = \frac{E_m}{\mu} \cos x \cos \hat{a}_z \quad v/m \ .$ 

- c. A parallel plate capacitor with plate area 5 cm<sup>2</sup> and plate separation of 3 mm has a voltage of 50 sin  $10^3$  t volts applied to its plates. Calculate the displacement current assuming  $\epsilon = 2 \epsilon_0$ .

  (08 Marks)
- 7 a. For an electromagnetic wave propagating in free space prove that  $\frac{|\vec{E}|}{|\vec{H}|} = \eta$ . (08 Marks)
  - b. State and explain Poynting's theorem. (06 Marks)
  - c. Calculate intrinsic impedance η, propagation constant γ and wave velocity ν for a conducting medium in which σ = 58 MS/m, μ<sub>r</sub> = 1, ∈<sub>r</sub> = 1 at frequency of 100 MHz.
     (06 Marks)
- 8 a. Define standing wave ratio. What is its relationship with the reflection co-efficient?
  (08 Marks)
  - b. A uniform plane wave of 200 MHz travelling in a free space impinges normally on a large block of material having  $\epsilon_r = 4$ ,  $\mu_r = 9$ ,  $\sigma = 0$ . Calculate transmission and reflection coefficients at the interface.
  - c. With usual notations, obtain the general wave equations for electric and magnetic fields.
    (06 Marks)

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## Third Semester B.E. Degree Examination, December 2011

### **Advanced Mathematics - I**

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

1 a. Express 
$$\frac{1}{(2+i)^2} - \frac{1}{(2-i)^2}$$
 in the form  $a + ib$ . (06 Marks)

b. Find the modulus and amplitude of 
$$\frac{(3-\sqrt{2}i)^2}{1+2i}$$
. (07 Marks)

c. Find the real part of 
$$\frac{1}{1+\cos\theta+i\sin\theta}$$
. (07 Marks)

a. Find the n<sup>-1</sup> derivative of 
$$\cos x \cos 2x \cos 3x$$
.  
b. If  $y = (\sin^{-1} x)^2$ , show that  $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$ . (07 Marks)

c. Find the nth derivative of 
$$\frac{x+2}{x+1} + \log\left(\frac{x+2}{x+1}\right)$$
. (07 Marks)

b. Given 
$$u = \sin\left(\frac{x}{y}\right)$$
,  $x = e^t$ ,  $y = t^2$ , find  $\frac{du}{dt}$  as a function of t. (07 Marks)

c. If 
$$x = r \cos \theta$$
,  $y = r \sin \theta$ , find  $\frac{\partial(x, y)}{\partial(r, \theta)}$  and  $\frac{\partial(r, \theta)}{\partial(x, y)}$ . (07 Marks)

4 a. Find the angle of intersection of the curves 
$$r = a(1 + \cos \theta)$$
 and  $r = b(1 - \cos \theta)$ . (06 Marks)

b. Find the pedal equation of the curve 
$$\frac{2a}{r} = 1 - \cos \theta$$
. (07 Marks)

c. Expand 
$$e^{\sin x}$$
 by Maclaurin's series upto the term containing  $x^4$ . (07 Marks)

5 a. Obtain the reduction formula for 
$$I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx$$
 where n is a positive integer. (06 Marks)

b. Evaluate: 
$$\int_{1}^{5} \int_{1}^{2} x(x^2 + y^2) dx dy$$
. (07 Marks)  
c. Evaluate:  $\int_{0}^{1} \int_{0}^{2} x^2 yz dx dy dz$ . (07 Marks)

c. Evaluate: 
$$\iint_{0}^{2} \iint_{0}^{2} x^2 yz \, dx \, dy \, dz.$$
 (07 Marks)

### MATDIP301

6 a. Prove that 
$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$
.

(06 Marks)

b. Show that 
$$\Gamma(n) = \int_0^1 \left(\log \frac{1}{x}\right)^{n-1} dx$$
.

(07 Marks)

c. Express 
$$\int_{0}^{\frac{\pi}{2}} \sqrt{\tan \theta} \ d\theta$$
 in terms of Gamma function.

(07 Marks)

7 a. Solve: 
$$\frac{dy}{dx} = \frac{x(2 \log x + 1)}{\sin y + y \cos y}$$
.

(06 Marks)

b. Solve: 
$$(1 + e^{x/y})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$$
.

(07 Marks)

c. Solve: 
$$(x^2 - ay)dx = (ax - y^2)dy$$
.

(07 Marks)

8 a. Solve: 
$$\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = 0$$
.

(06 Marks)

b. Solve: 
$$(D-2)^2 y = 8(e^{2x} + \sin 2x)$$
.

(07 Marks)

c. Solve: 
$$(D^3 + 4D)y = \sin 2x$$
.

(07 Marks)